6. Halla los siguientes límites utilizando la regla de L`Hôpital:

a) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{3x^2 - 2x}$ **b)** $\lim\limits_{x \to +\infty} \dfrac{e^x - 1}{\ln(e^x + 1)}$ **c)** $\lim\limits_{x \to 0} \dfrac{2x - 3\text{sen}x + x\cos x}{x^5}$

d) $\lim\limits_{x \to 1} \dfrac{\ln x}{3x^3 - 3}$ **e)** $\lim\limits_{x \to 0} \cos x^{1/\text{sen}^2 x}$ **f)** $\lim\limits_{x \to 0} \left(\dfrac{1}{\text{sen}^2 x} - \dfrac{1}{x^2}\right)$

Solución

a) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{3x^2 - 2x} = \left[\dfrac{0}{0}\right] \underset{\text{(L'Hôpital)}}{=} \lim\limits_{x \to 0} \dfrac{e^x}{6x - 2} = \dfrac{1}{-2} = -\dfrac{1}{2}$

b) $\lim\limits_{x \to +\infty} \dfrac{e^x - 1}{\ln(e^x + 1)} = \left[\dfrac{+\infty}{+\infty}\right] \underset{\text{(L'Hôpital)}}{=} \lim\limits_{x \to +\infty} \dfrac{e^x}{\dfrac{e^x}{e^x + 1}} = \lim\limits_{x \to +\infty} \dfrac{1}{\dfrac{1}{e^x + 1}} = \lim\limits_{x \to +\infty} \left(e^x + 1\right) = +\infty$

c) $\lim\limits_{x \to 0} \dfrac{2x - 3\text{sen}x + x\cos x}{x^5} = \left[\dfrac{0}{0}\right] \underset{\text{(L'Hôpital)}}{=} \lim\limits_{x \to 0} \dfrac{2 - 3\cos x + \cos x - x\,\text{sen}x}{5x^4} = \lim\limits_{x \to 0} \dfrac{2 - 2\cos x - x\,\text{sen}x}{5x^4} =$

$= \left[\dfrac{0}{0}\right] \underset{\text{(L'Hôpital)}}{=} \lim\limits_{x \to 0} \dfrac{2\text{sen}x - \text{sen}x - x\cos x}{20x^3} = \lim\limits_{x \to 0} \dfrac{\text{sen}x - x\cos x}{20x^3} = \left[\dfrac{0}{0}\right] \underset{\text{(L'Hôpital)}}{=}$

$= \lim\limits_{x \to 0} \dfrac{\cos x - \cos x + x\,\text{sen}x}{60x^2} = \lim\limits_{x \to 0} \dfrac{x\,\text{sen}x}{60x^2} = \left[\dfrac{0}{0}\right] \underset{\text{(L'Hôpital)}}{=} \lim\limits_{x \to 0} \dfrac{\text{sen}x + x\cos x}{120x} = \left[\dfrac{0}{0}\right] \underset{\text{(L'Hôpital)}}{=}$

$= \lim\limits_{x \to 0} \dfrac{\cos x + \cos x - x\,\text{sen}x}{120} = \lim\limits_{x \to 0} \dfrac{2\cos x + x\,\text{sen}x}{120} = \dfrac{2}{120} = \dfrac{1}{60}$

d) $\lim\limits_{x \to 1} \dfrac{\ln x}{3x^3 - 3} = \left[\dfrac{0}{0}\right] \underset{\text{(L'Hôpital)}}{=} \lim\limits_{x \to 1} \dfrac{\frac{1}{x}}{9x^2} = \lim\limits_{x \to 1} \dfrac{1}{9x^3} = \dfrac{1}{9}$

e) $\lim\limits_{x \to 0} \cos x^{1/\text{sen}^2 x} = \left[1^{+\infty}\right] = e^{\lim\limits_{x \to 0} \frac{1}{\text{sen}^2 x}(\cos x - 1)} = e^{\lim\limits_{x \to 0} \frac{\cos x - 1}{\text{sen}^2 x}} = e^{\left[\frac{0}{0}\right]} \underset{\text{(L'Hôpital)}}{=} e^{\lim\limits_{x \to 0} \frac{-\text{sen}x}{2\text{sen}x\cos x}} =$

$= e^{\lim\limits_{x \to 0} \frac{-1}{2\cos x}} = e^{\frac{-1}{2}} = \dfrac{1}{\sqrt{e}}$

f) $\lim\limits_{x \to 0} \left(\dfrac{1}{\text{sen}^2 x} - \dfrac{1}{x^2}\right) = \left[+\infty - \infty\right] = \lim\limits_{x \to 0} \left(\dfrac{x^2 - \text{sen}^2 x}{x^2 \text{sen}^2 x}\right) = \left[\dfrac{0}{0}\right]$, en este caso si aplicamos directamente la regla de L'Hôpital el proceso no se acaba. Por ello vamos a sustituir la equivalencia "$\text{sen}x \sim x$ si $x \to 0$" cuando $\text{sen}x$ aparezca como factor, es decir, en el denominador, quedando

$\lim\limits_{x \to 0} \left(\dfrac{x^2 - \text{sen}^2 x}{x^2 x^2}\right) = \lim\limits_{x \to 0} \left(\dfrac{x^2 - \text{sen}^2 x}{x^4}\right) = \left[\dfrac{0}{0}\right]$ y ahora se resuelve por la regla de L'Hôpital

$$\lim_{x\to 0}\left(\frac{x^2-\text{sen}^2 x}{x^4}\right)=\left[\frac{0}{0}\right]\underset{\text{(L'Hôpital)}}{=}\lim_{x\to 0}\left(\frac{2x-2\text{sen}x\cos x}{4x^3}\right)\underset{\text{sen}2x=2\text{sen}x\cos x}{=}\lim_{x\to 0}\left(\frac{2x-\text{sen}2x}{4x^3}\right)=$$

$$=\left[\frac{0}{0}\right]\underset{\text{(L'Hôpital)}}{=}\lim_{x\to 0}\left(\frac{2-2\cos 2x}{12x^2}\right)==\left[\frac{0}{0}\right]\underset{\text{(L'Hôpital)}}{=}\lim_{x\to 0}\left(\frac{4\text{sen}2x}{24x}\right)=\left[\frac{0}{0}\right]\underset{\text{(L'Hôpital)}}{=}\lim_{x\to 0}\left(\frac{8\cos 2x}{24}\right)=$$

$$=\frac{8}{24}=\frac{1}{3}$$