

Razones trigonométricas de la suma/diferencia de dos ángulos

SUMA

$$\operatorname{sen}(\alpha + \beta) = \operatorname{sen}\alpha \cos\beta + \operatorname{cos}\alpha \operatorname{sen}\beta$$

$$\operatorname{cos}(\alpha + \beta) = \operatorname{cos}\alpha \operatorname{cos}\beta - \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \operatorname{tg}\beta}$$

DIFERENCIA

$$\operatorname{sen}(\alpha - \beta) = \operatorname{sen}\alpha \operatorname{cos}\beta - \operatorname{cos}\alpha \operatorname{sen}\beta$$

$$\operatorname{cos}(\alpha - \beta) = \operatorname{cos}\alpha \operatorname{cos}\beta + \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha \operatorname{tg}\beta}$$

Ejemplo 11: Calcular las razones de 75° y de 15° en función de las de 45° y 30°

$$\operatorname{sen}75^\circ = \operatorname{sen}(45^\circ + 30^\circ) = \operatorname{sen}45^\circ \operatorname{cos}30^\circ + \operatorname{cos}45^\circ \operatorname{sen}30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\operatorname{cos}75^\circ = \operatorname{cos}(45^\circ + 30^\circ) = \operatorname{cos}45^\circ \operatorname{cos}30^\circ - \operatorname{sen}45^\circ \operatorname{sen}30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\operatorname{tg}75^\circ = \operatorname{tg}(45^\circ + 30^\circ) = \frac{\operatorname{tg}45^\circ + \operatorname{tg}30^\circ}{1 - \operatorname{tg}45^\circ \operatorname{tg}30^\circ} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\operatorname{sen}15^\circ = \operatorname{sen}(45^\circ - 30^\circ) = \operatorname{sen}45^\circ \operatorname{cos}30^\circ - \operatorname{cos}45^\circ \operatorname{sen}30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\operatorname{cos}15^\circ = \operatorname{cos}(45^\circ - 30^\circ) = \operatorname{cos}45^\circ \operatorname{cos}30^\circ + \operatorname{sen}45^\circ \operatorname{sen}30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\operatorname{tg}15^\circ = \operatorname{tg}(45^\circ - 30^\circ) = \frac{\operatorname{tg}45^\circ - \operatorname{tg}30^\circ}{1 + \operatorname{tg}45^\circ \operatorname{tg}30^\circ} = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$